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BRIEF COMMUNICATIONS AND LETTERS TO THE EDITOR
ON THE PROPAGATION OF ELASTIC WAVES IN PIEZOSEMICONDUCTORS

V. V. Soroka

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ON THE PROPAGATION OF ELASTIC WAVES IN PIEZOSEMICONDUCTORS

V. V. Soroka

ABSTRACT. The general invariant expressions for the velocity and decay of elastic waves in piezoelectric media are derived. The study is based on joint solution of the Maxwell equations and equations of motion of the piezoelectric medium.

This communication presents a derivation of the general invariant expressions for the velocity and decay of elastic waves in piezoelectric media for arbitrary propagation directions, specified by the unit vector n with the components n_i . The properties of the medium are described by the arbitrary tensors of the electrical conductivity σ_{ik} , elastic λ_{iklm} , piezoelectric β_{ikl} , and dielectric ϵ_{ik} constants ($i, k, l, m = 1, 2, 3$). We neglect the magnetic properties and the viscosity of the medium. Therefore, the wave decay is due only to the interaction of the electric field, which accompanies the elastic wave in a piezoelectric, with the free carriers. The study is made within the framework of phenomenological theory and is based on joint solution of the Maxwell equations and the equations of motion of the piezoelectric medium. We seek the solution in the form of plane waves. Then we obtain the tensor equation for the elastic wave displacement vector u .

$$\left\{ \kappa^2 \Lambda - \omega^2 + \frac{4\pi\kappa^2}{\rho} f \left(\frac{c^2\kappa^2}{\omega^2} n^2 + \epsilon \right)^{-1} f \right\} u = 0, \quad (1)$$

where $k = kn$ is the wave vector; f is a second-rank tensor with the components $f_{ik} = \beta_{ikl} n_l$; f is a tensor conjugate with f ; Λ is a second-rank symmetric tensor with the components $\Lambda_{ik} = \frac{1}{p} \lambda_{ilk m} n_l n_m$; p is the density of the medium; c is the speed of light in vacuo.

Moreover, n^x is an antisymmetric tensor which is dual to the vector n [1], and the complex dielectric permeability tensor ϵ has the components:

Equating the determinant of the tensor for u to zero, we obtain the most general dispersion equation for elastic waves in piezoelectric media:

$$\left| \kappa^2 \Lambda - \omega^2 + \frac{4\pi\kappa^2}{\rho} \tilde{f} \left(\frac{c^2\kappa^2}{\omega^2} n^x + \epsilon \right)^{-1} f \right| = 0. \quad (2)$$

Considering that the elastic wave velocity $v = \frac{\omega}{k} < c$, and neglecting terms of order $\frac{v^2}{c}$, we have:

$$\left(\frac{c^2\kappa^2}{\omega^2} n^x + \epsilon \right)^{-1} = \frac{n \cdot n}{\epsilon_n}, \quad (3)$$

where $n \cdot n$ is a tensor-dyad with the components $(n \cdot n)_{ik} = n_i n_k$, and ϵ_n is the value of the complex dielectric permeability in the direction n , equal to [2]:

$$\epsilon_n = n \epsilon n. \quad (4)$$

Then (1) and (2) may be simplified:

$$\left\{ \kappa^2 \Lambda - \omega^2 + \frac{4\pi\kappa^2}{\rho} \frac{n f \cdot n f}{\epsilon_n} \right\} u = 0; \quad (5)$$

$$\left| \kappa^2 \Lambda - \omega^2 + \frac{4\pi\kappa^2}{\rho} \frac{n f \cdot n f}{\epsilon_n} \right| = 0. \quad (6)$$

The dispersion Equation (6) breaks down into two equations if we equate the imaginary and real parts of the expression appearing in it to zero separately, using the formula for the determinant of the tensor sum [2].

$$|\beta + \gamma| = |\beta| + (\bar{\beta}\gamma)_c + (\bar{\gamma})_c + |\gamma|. \quad (7)$$

The following notations are used in (7): $\bar{x}$ is the tensor reciprocal to x , $|x|$ is the determinant, and x_c the trace of the tensor x .

Then, we consider that for most dielectrics $\text{Im} \left(\frac{\omega^2}{k} \right) \ll \text{Re} \left(\frac{\omega^2}{k} \right)$ and we obtain /130 the general invariant expressions for the phase velocity and decay of elastic waves in piezosemiconductors:

$$v = v_0 + \frac{2\pi \text{Re} \epsilon_n}{\rho v_0 |\epsilon_n|^2} A; \quad (8)$$

$$\alpha = \text{Im} \kappa = \frac{2\pi\omega}{\rho v_0^3} \frac{\text{Im} \epsilon_n}{|\epsilon_n|^2} A; \quad (9)$$

$$A = \frac{nf(\Lambda - v_0^2)nf}{(\Lambda - v_0^2)_c}; \quad (10)$$

v_0 is the velocity of elastic waves in the medium in question without account for the influence of the piezoelectric effect, which may be determined from the Christoffel equation [2].

Formula (9) for the piezoelectric decay can also be obtained from the Joule-Lenz law, if we know the general expression for the intensity E of the electric field which accompanies the elastic wave with displacement vector u in a piezoelectric medium. The calculations lead to the invariant expression:

$$E = 4\pi i \kappa \frac{nf u}{\epsilon_n} n. \quad (11)$$

Then, with account for (11) and the Joule-Lenz law we obtain:

$$\alpha = \frac{2\pi\omega}{\rho v_0^3} \frac{\text{Im} \epsilon_n}{|\epsilon_n|^2} \frac{(nf u)^2}{u^2}. \quad (12)$$

With the aid of (5), it is not difficult to show that (12) and (9) coincide, if we neglect the influence of the piezoeffect on the wave velocity.

In the particular case, when the wave is purely longitudinal, i.e., $u \parallel n$:

$$\alpha = \frac{2\pi\omega}{\rho v_0^3} \frac{\text{Im} \epsilon_n}{|\epsilon_n|^2} (nfn)^2. \quad (13)$$

Expression (13) coincides with the analogous formula of Pustovoyt [3]. However, in the general case, when the propagation direction does not coincide with the longitudinal normal of the crystal, the wave is not purely longitudinal and therefore (13) becomes inapplicable.

Let us examine (10), which appears in (8) and (9), for certain particular cases.

1. Sphalerite. If the propagation direction coincides with one of the coordinate axes, then $A = 0$ for both longitudinal and shear waves

($A_{\parallel} = A_{\perp} = A''_{\perp} = 0$). If the wave propagates along the (110) direction in the crystal, then $A_{\parallel} = A'_{\perp} = 0$; $A''_{\perp} = \beta_{123}^2$.

2. Wurtzite. For propagation along the z axis (001) $A'_{\perp} = A''_{\perp} = 0$; $A_{\parallel} = \beta_{333}^2$, while in the plane perpendicular to z, $A = A' = 0$; $A'' = \beta_{113}^2$.

3. Quartz. If n coincides with (100), then $A'_{\perp} = A''_{\perp} = 0$; $A_{\parallel} = \beta_{111}^2$.

Then from (8) and (9) we obtain expressions for v and a, which coincide with the analogous expressions of [4]. For wave propagation along the optical axis of the quartz $A'_{\perp} = A''_{\perp} = A_{\parallel} = 0$.

In all the considered examples, the directions of the coordinate axes are taken in accordance with [5].

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Leningrad Institute of Aviation .

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